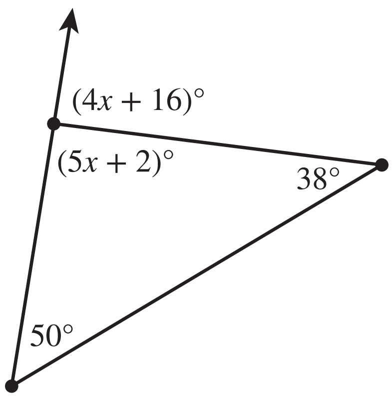
LESSON 16

Name ____________________ Date __________

Find Unknown Angle Measures

1. Use the diagram shown.



a. Find the value of x.

$$(5x + 2) + 50 + 38 = 180$$
$$5x + 90 = 180$$
$$5x = 90$$
$$x = 18$$

The value of x is 18.

b. What angle relationship did you use for part (a)?

The interior angle measures of a triangle sum to 180°.

Angle Relationship Search

For each card,

- describe all angle relationships used, and
- write and solve an equation to support your answer.

2. Angle relationships:

   The interior angle measures of a triangle sum to $180°$.

   Equation:

$$\begin{aligned} 39° + 90° + m\angle 1 &= 180° \\ 129° + m\angle 1 &= 180° \\ m\angle 1 &= 51° \end{aligned}$$

3. Angle relationships:

   When lines cut by a transversal are parallel, the measures of the corresponding angles are equal. Angles in a linear pair have measures that sum to $180°$.

   Equation:

$$m\angle 2 = 62.5°$$

$$\begin{aligned} m\angle 1 + m\angle 2 &= 180° \\ m\angle 1 + 62.5° &= 180° \\ m\angle 1 &= 117.5° \end{aligned}$$

4. Angle relationships:

   Supplementary angles have measures that sum to $180°$. The sum of the interior angle measures of a triangle is $180°$.

   Equation:

$$\begin{aligned} 180° - 110° &= m\angle 1 \\ 70° &= m\angle 1 \end{aligned}$$

$$\begin{aligned} 180° - 154° &= m\angle 3 \\ 26° &= m\angle 3 \end{aligned}$$

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 &= 180° \\ 70° + m\angle 2 + 26° &= 180° \\ 96° + m\angle 2 &= 180° \\ m\angle 2 &= 84° \end{aligned}$$

5. Angle relationships:

The interior angle measures of a triangle sum to 180°.

Equation:

$$\begin{aligned} 2x + 2x + 2x &= 180 \\ 6x &= 180 \\ x &= 30 \end{aligned}$$

6. Angle relationships:

When lines cut by a transversal are parallel, the measures of the corresponding angles are equal.

Equation:

$$\begin{aligned} 2x + 7 &= 77 \\ 2x &= 70 \\ x &= 35 \end{aligned}$$

7. Angle relationships:

   The exterior angle measure of a triangle equals the sum of the remote interior angle measures.

   Equation:

$$72 + 63 = 8x + 75$$
$$135 = 8x + 75$$
$$60 = 8x$$
$$7.5 = x$$

8. Angle relationships:

   The exterior angle measure of a triangle equals the sum of the remote interior angle measures.
   Supplementary angles have measures that sum to 180°.

   Equation:

$$54 + 75 = 2y - 7$$
$$129 = 2y - 7$$
$$136 = 2y$$
$$68 = y$$

$$54 + (4x - 12) = 180$$
$$4x + 42 = 180$$
$$4x = 138$$
$$x = 34.5$$

$$68 + 34.5 = 102.5$$

9. Angle relationships:

When lines cut by a transversal are parallel, the measures of the alternate exterior angles are equal. Angles in a linear pair have measures that sum to 180°.

Equation:

$$5x - 11 = 53$$
$$5x = 64$$
$$x = 12.8$$

$$(10y + 7) + (5x - 11) = 180$$
$$(10y + 7) + (5(12.8) - 11) = 180$$
$$(10y + 7) + (64 - 11) = 180$$
$$10y + 7 + 53 = 180$$
$$10y + 60 = 180$$
$$10y = 120$$
$$y = 12$$

$$12.8 + 12 = 24.8$$

10. Angle relationships:

Supplementary angles have measures that sum to 180°. The interior angle measures of a triangle sum to 180°.

Equation:

$$\begin{aligned}(3y-2)+101&=180\\3y+99&=180\\3y&=81\\y&=27\end{aligned}$$

$$\begin{aligned}(4x+7)+(3y-2)+42&=180\\(4x+7)+(3(27)-2)+42&=180\\(4x+7)+(81-2)+42&=180\\(4x+7)+79+42&=180\\4x+128&=180\\4x&=52\\x&=13\end{aligned}$$

$$27+13=40$$

EXIT TICKET 16

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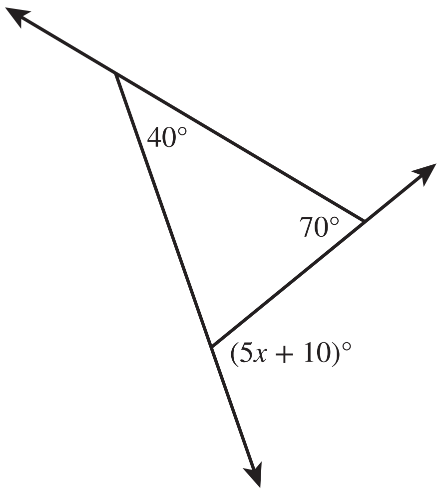
Consider the diagram.



a. Find the value of x.

$$40 + 70 = 5x + 10$$
$$110 = 5x + 10$$
$$100 = 5x$$
$$20 = x$$

The value of x is 20.

b. Describe the angle relationship you used to find the value of x.

The exterior angle measure of a triangle is equal to the sum of the remote interior angle measures.

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RECAP 16

Find Unknown Angle Measures

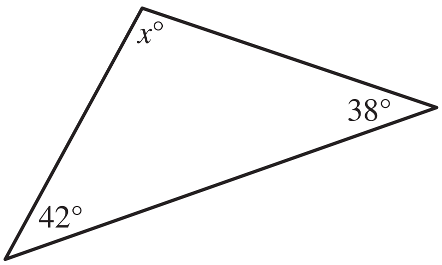
In this lesson, we

- determined the angle relationships in a given diagram.
- wrote equations by using angle relationships to find unknown values.

Examples

For problems 1 and 2, write an equation and find the value of x. Describe all angle relationships you use to write the equation.

1.



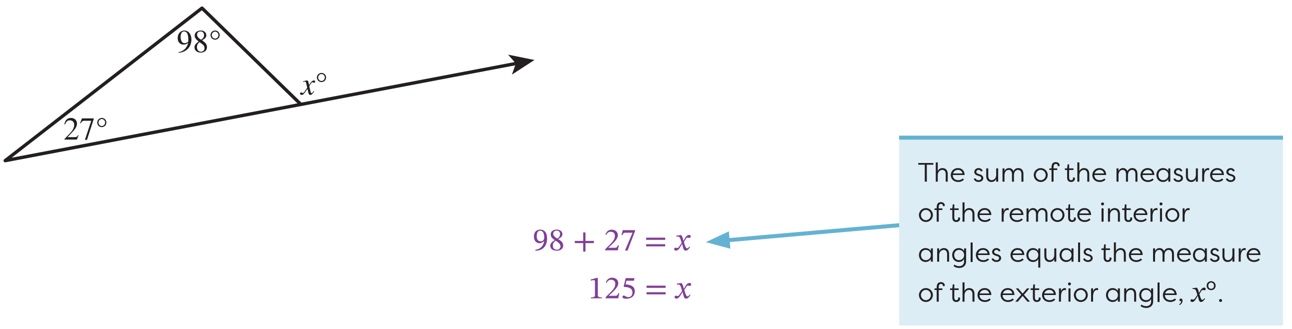
$$x + 42 + 38 = 180$$
$$x + 80 = 180$$
$$x = 100$$

Write an equation that shows the sum of all three interior angle measures equals 180°. Then solve for x.

The value of x is 100.

The interior angle measures of a triangle sum to 180°.

2.

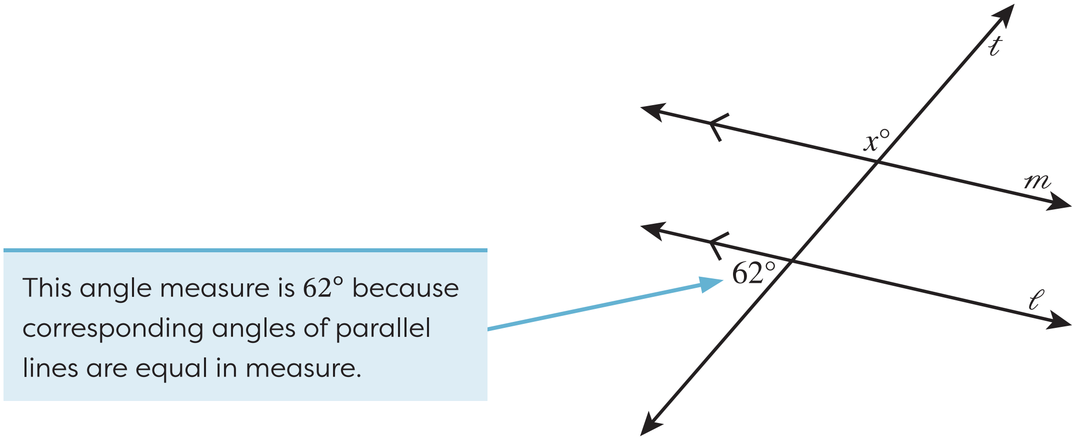


$$98 + 27 = x$$
$$125 = x$$

The value of x is 125.

The exterior angle measure of a triangle is equal to the sum of the remote interior angle measures.

3. Parallel lines ℓ and m are cut by transversal t. Write an equation and find the value of x. Identify all angle relationships you use to write the equation.



$$62 + x = 180$$
$$x = 118$$

The value of x is 118.

The measures of the corresponding angles are equal because lines ℓ and m are parallel. Linear pairs have measures that sum to 180°.

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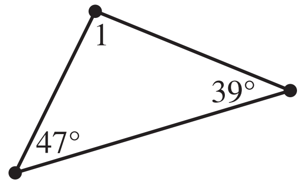
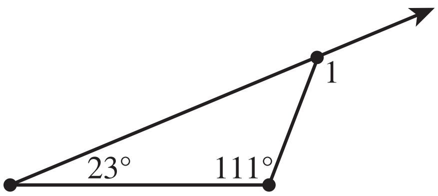
PRACTICE 16

For problems 1–3, write an equation by using angle relationships given in the diagram. Then find the measure of ∠1.

1. 

$m\angle 1 + 47° + 39° = 180°$

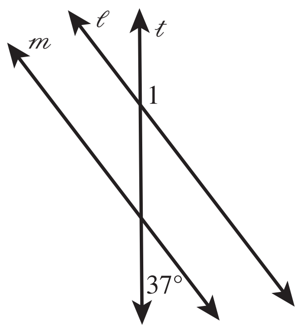
94°

2. 

$23° + 111° = m\angle 1$

134°

3. Assume $\ell \parallel m$.

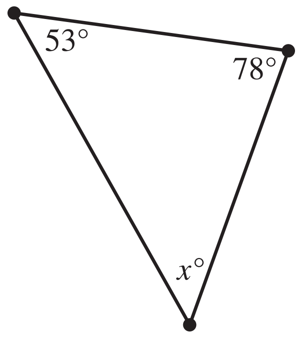


$180° - 37° = m\angle 1$

143°

For problems 4–6, write an equation by using angle relationships given in the diagram. Then find the value of x.

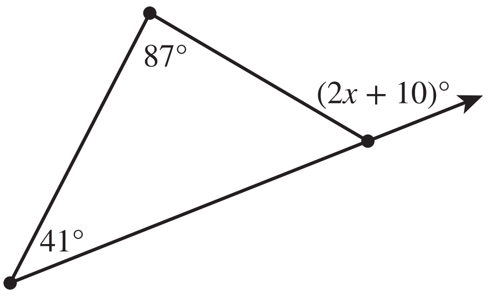
4.



$53 + 78 + x = 180$

The value of x is 49.

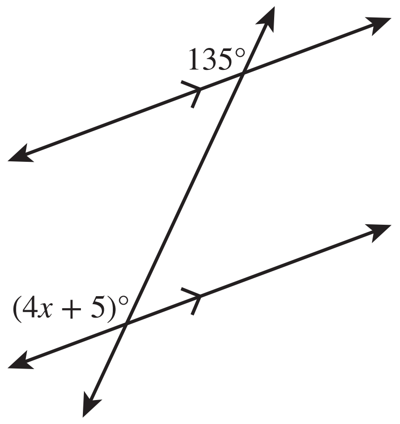
5.



$2x + 10 = 87 + 41$

The value of x is 59.

6.

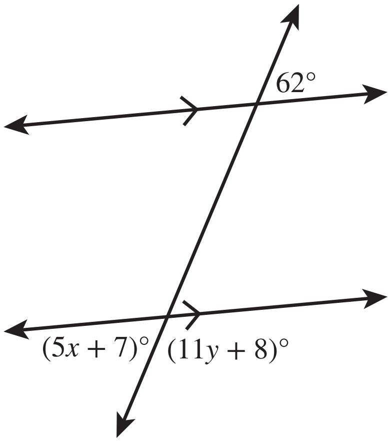


$4x + 5 = 135$

The value of x is 32.5.

For problems 7 and 8, write equations by using angle relationships given in the diagram. Then find the value of x and y.

7.



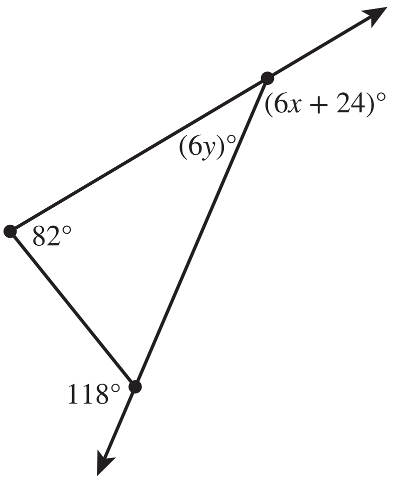
$5x + 7 = 62$

The value of x is 11.

$11y + 8 + 62 = 180$

The value of y is 10.

8.



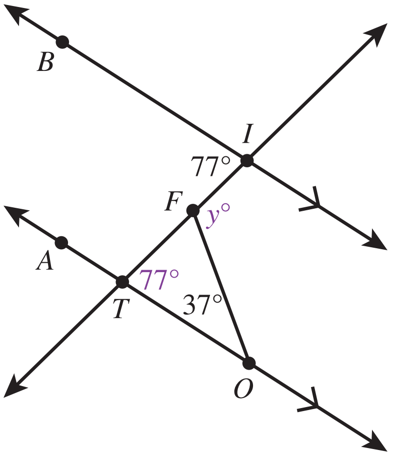
$118 = 82 + 6y$

The value of y is 6.

$(6x + 24) + (6y) = 180$

The value of x is 20.

9. Ava solves this problem correctly. Analyze Ava's work. Then find the measure of $\angle IFO$ by using a different strategy. Explain your work.



Ava's Work:

$$77 + 37 = y$$
$$114 = y$$

Because $\overleftrightarrow{BI} \parallel \overleftrightarrow{AO}$, alternate interior angles $\angle BIT$ and $\angle FTO$ are congruent. So the measure of $\angle FTO$ is 77°. The measure of the exterior angle of the triangle, $\angle IFO$, is equal to the sum of the measures of the remote interior angles, $\angle FTO$ and $\angle TOF$. So the measure of $\angle IFO$ is 114°.

Let $m\angle TFO = x°$ and $m\angle IFO = y°$.

$$77 + 37 + x = 180$$
$$114 + x = 180$$
$$x = 66$$

$$x + y = 180$$
$$66 + y = 180$$
$$y = 114$$

The measure of $\angle IFO$ is 114°.

Remember

For problems 10–13, write an equivalent expression.

10. $5(x+3)+8(x+2)$

    $13x+31$

11. $9(x-5)+3(x-2)$

    $12x-51$

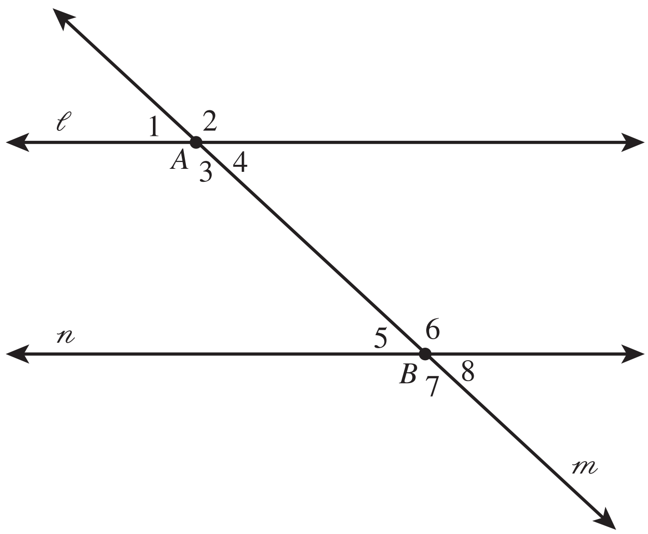
12. $-2(x-4)+4(x+2)$

    $2x+16$

13. $3(x-4)-7(x-1)$

    $-4x-5$

14. Parallel lines ℓ and n are cut by transversal m.



a. In the diagram, $\angle 2$ and $\angle 6$ have what angle relationship?

   They are corresponding angles.

b. Use rigid motions to describe how you know $\angle 2$ and $\angle 6$ are congruent.

   A translation along $\overrightarrow{AB}$ maps $\angle 2$ onto $\angle 6$, so the angles are congruent.

15. Consider the equation $\frac{27 \cdot 9^2}{3^n} = 3^6$. What is the value of n?

    1